Integrated Pool - Bilateral and Reserve Electricity Markets through Pay-as-Bid Pricing

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Abstract—**This paper presents a pricing model considering the simultaneous interaction of pool, bilateral and reserve markets in a power system. The model is adapted to work through a 'pay as bid' (PAB) pricing approach that is currently being considered as an option for pricing in some actual systems. The combined market is settled through an AC Optimal Power Flow (OPF) that accounts for power generation (pool and bilateral contracts) and availability of reserve services. As a result, prices of energy and reserve services incorporate the influences of the transmission network such as topology, voltage levels, losses, generation and transmission capacity limits. Results show that with this model agents can better plan their portfolios based on prices that reflect the costs of resources considering several operation scenarios and bid strategies.**

Index Terms— **Optimal Power Flow, Pay as bid, Pool, Bilateral and Reserve Markets.**

I. INTRODUCTION

IN recent years, electricity markets have developed important economic and operational tools looking for efficiency in terms of determining prices that represent the costs of production and transmission of energy. This effort has been oriented to obtain corresponding payments and revenues that truly reflect the use of services and provide economic signals for future investments in infrastructure. One important aspect in pricing is from the structural point of view of the main two existing forms of markets auctions for trading energy and services [1]. One is based on private markets and the other is based on a strong coordination. In the first one, the provision of services is left to secondary markets after the main energy auction is defined. In this structure energy and services are provided through an unbundled mechanism. Because of the free participation of traders in each market for each service, opportunity for arbitrage motivates traders to move from one market to another. The possible advantage of this auction structure is that the voluntary participation of traders in markets could provide efficiency, like in a pool, avoiding using complex optimization tools. In the second kind of market auctions, market products are procured simultaneously through central auctions. The purpose of this type of auction is that optimization is necessary to minimize costs of generation, transmission and reserves to meet demand and ensure reliability. The advantage of this integrated market is that the resulting pricing better reflects the cost of resources. Several systems operate with this structure like in New York, New England and PJM. Also, several models have been suggested following this trend.

From a theoretical point of view, two approaches are mainly followed for pricing electricity services: One of them is the classical marginal pricing (MP) where nodal prices represent the cost of the last MW to be supplied and the other is "pay as bid" pricing (PAB) which is the way forward bilateral contracts are negotiated. Whether one or the other should be followed has been object of controversy and it is not the focus of this paper. Recently, PAB pricing has showed an increasing interest because some markets like in Wales and England are essentially based on bilateral agreements that are traded following a PAB approach. Additionally, new studies have shown some advantages of PAB over MP. For instance, in [14] it is demonstrated that in the general case when the load is not exactly predicted, the expected profits and load payments are the same under MP and PAB but the risk of not meeting these values are greater under MP.

This paper analyzes the characteristics of a new pricing model designed for working under PAB or MP approaches in a combined integrated market structure with the presence of forward bilateral contracts. The formulation of the model is similar to the one presented in [4] but with the difference of considering the ancillary reserve market as well. This strategy allows studying the implications in the operation and economic indexes (such as revenues and payments portfolios and prices) in an integrated market including energy and ancillary services. In the integrated market the awarded obtained levels and prices of energy and ancillary reserve services not only reflect the impact of their mutual interaction but also the impact due to the coexistence with the forward bilateral contract market. The suggested pricing model incorporates the previously mentioned advantages of a centralized coordination and the PAB pricing strategy.

The model considers the ancillary reserve market composed of several reserve services as in [1,2,15]. The characterization of these services is previously defined by the regulator and is based on the quality (speed) of their response. The possibility of substitution among reserve services based on agent bids avoids price reversal between them as considered in [1,15].

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The PAB version of this model is implemented through an integration process based on the Auman-Shapley [3,4] technique that takes into consideration the non-linear characteristic existing in the transmission network due to transmission losses and voltage behaviors using an AC - OPF. These features are not present in simpler PAB schemes such as pro-rata. The integration process allows also to unbundling the use and prices of several services including transmission bilateral contracts losses, pool dispatch, and availability of reserve services.

One of the main advantages of this approach is the fact that market agents can better plan their portfolios of products to increase revenues (generators) or minimize payments (loads) because of the possibility of considering different operation scenarios with a less risky pricing approach such as PAB. From the ISO point of view, it is possible to estimate the impact of different levels of total bilateral trades, generation and transmission capacity and reserve requirements on the operation and markets. A detailed mathematical formulation of the problem is presented showing how prices of energy and reserve ancillary services can be obtained from Lagrangean multipliers.

The paper is organized as follows. Section 3 presents the formulation used to represent the integrated market and bilateral contracts as well as the pricing mechanism. Section 4 presents numerical examples and Section 5 presents the conclusions.

II. NOMENCLATURE

For each generator *i* participating in the integrated market we define the following notation.

Functions:

$C_{g\,i}(p_{g\,i})$	Energy bid ($/h)
$C_{g\,i}^{RU}(ru_{g\,i})$	Regulation up bid ($/h)
$C_{g\,i}^{RD}(rd_{g\,i})$	Regulation down bid ($/h)
$C_{g\,i}^{SR}(sr_{g\,i})$	Spinning reserve bid ($/h)
$C_{g\,i}^{NS}(ns_{g\,i})$	Non spinning reserve bid ($/h)
$C_{g\,i}^{RC}(rc_{g\,i})$	Complementary reserve bid ($/h)

Variables:

$p_{g\,i}$	Total production of Generator i (MW)
$p_{g\,i}^{\,p}$	Pool Generation awarded level (MW)
$ru_{\,g\,i}$	Awarded level of regulation up reserve (MW)
$rd_{\,g\,i}$	Awarded level of regulation down reserve (MW)
$sr_{g\,i}$	Awarded level of spinning reserve (MW)
$ns_{\,g\,i}$	Awarded level of non spinning reserve (MW)
$rc_{\,g\,i}$	Awarded level of complementary reserve (MW)
$q_{\,g\,i}$	Reactive power generation level (MVar)
v_i , δ_i	Bus *i* voltage phasor module and angle

Constant Parameters:

$p_{d\,j}$	Active power demand level at bus *j* (MW)
R^{RU}	Amount of regulation up reserve required by the system (MW)
R^{RD}	Amount of regulation down reserve required by the system (MW)
R^{SR}	Amount of spinning reserve required by the system (MW)
R^{NS}	Amount of non spinning reserve required by the system (MW)
R^{RC}	Amount of complementary reserve required by the system (MW)
N	Number of integration steps
$RP_{g\,i}^{RU}$	10 min. ramp rate for providing $ru_{\,g\,i}$ in MW/minute
$RP_{g\,i}^{SNS}$	10 min. ramp rate for providing $sr_{g\,i}$ and $ns_{\,g\,i}$ in MW/minute

Lagrange Multipliers (LM):

λ_i^p	Active power nodal Price at bus *i* ($/MWh)
λ_i^q	Reactive power nodal price at bus *i* ($/MVarh)
λ^{RU}	LM of regulation up requirement ($/MWh)
λ^{RD}	LM of regulation down requirement ($/MWh)
λ^{SR}	LM of spinning reserve requirement ($/MWh)
λ^{NS}	LM of non spinning reserve requirement ($/MWh)
λ^{RC}	LM of complementary reserve requirement ($/MWh)

III. FORMULATION

The main idea is first formulating an integrated market of energy and reserve services and next characterizing a pricing model based on a PAB approach. Because of this, the model presented here considers that a Unit Commitment (UC) problem has been solved previously and the set of scheduled units are known for solving an AC OPF in the auction process. This procedure is followed in some power systems. In order to simplify the presentation, we consider that load has no participation in the ancillary service market by submitting curtailment bids in a load management program. Nevertheless, this extension can be easily incorporated as it was done in [2]. Also, we consider a one hour pricing model with no inter-temporal constrains and that enough support of reactive power is available to keep voltages close to nominal values.

A. Pool Market Model

Pool market determines awarded energy and reserve bids for a period of one hour. In this market low cost merit order lists for energy and reserve services are obtained by minimizing the following objective function.

$$
\begin{aligned}
Minimize\ \Big\{ & \sum_i C_i(p_{g\,i}) \rightarrow (energy) \\
& + \sum_i C_i^{RU}(ru_{g\,i}) \rightarrow (Regulation\ Up\ Reserve) \\
& + \sum_i C_i^{SR}(sr_{g\,i}) \rightarrow (Spinning\ \ Reserve) \\
& + \sum_i C_i^{NS}(ns_{g\,i}) \rightarrow (non-Spinning\ \ Reserve) \\
& + \sum_i C_i^{RC}(rc_{g\,i}) \rightarrow (Complementary\ \ Reserve) \\
& + \sum_i C_i^{RD}(rd_{g\,i}) \rightarrow (Regulation\ \ Down\ \ Reserve)\Big\}
\end{aligned} \quad (1)
$$

Cost bid functions can be considered as continuous quadratic functions approximations or piece-wise linear functions.

B. Transmission Network Model

From the ISO point of view, awarded bids and firm bilateral contracts must attend operation constraints imposed by the transmission system. Therefore, feasible solutions should belong to the set S defined by constraints (2) to (7). Equations (2) and (3) represent the load flow equations. Constraint (4) represents transmission line active flow limits, constraints (5) to (7) represent capacity limits and voltage limits. Vector $\mathbf{v}$ represents bus voltage modules and vector δ represents bus phase voltage angles. Both vectors have dimension equal to the number of buses *n*.

$$p_i = p_{g\,i} - p_{d\,i} = p_i(\mathrm{v},\delta), i = 1,...,n \rightarrow \lambda_i^p \quad (2)$$

$$q_i = q_{g\,i} - q_{d\,i} = q_i(\mathrm{v},\delta)\,, i = 1,...,n \rightarrow \lambda_i^q \quad (3)$$

$$-p_{i\,j}^{\max} \le p_{i\,j} \le p_{i\,j}^{\max}, \quad i, j = 1,...,n \quad (4)$$

$$p_{g\,i}^{\min} \le p_{g\,i} \le p_{g\,i}^{\max}, i = 1,...,n \quad (5)$$

$$q_{g\,i}^{\min} \le q_{g\,i} \le q_{g\,i}^{\max}, i = 1,...,n \quad (6)$$

$$v_i^{\min} \le v_i \le v_i^{\max}, i = 1,...,n \quad (7)$$

Lambda vectors in (2) and (3) represent Lagrange multipliers associated to each constraint and also represent nodal prices for active and reactive power.

C. Reserve Market characteristics

Generators agents can bid in five kinds of reserve services: Regulation up, Regulation Down, Spinning, Non-Spinning, and Replacement. The speed of response defines the quality of each service. Faster regulations services such as regulation up and down and spinning reserve are considered of higher quality. In order to open possibilities for reducing costs, it is also allowed the possibility of substitution among reserve services. The substitution consists in allowing that services with better quality and lower cost can substitute services with lower quality [1,15]. A feasible bid selection in the reserve market auction should belong to the set E described by constraints (8) to (14).

$$R^{RU} \le \sum_{i=1}^{n} ru_{g\,i} \rightarrow \lambda^{RU} \quad (8)$$

$$R^{RU} + R^{SR} \le \sum_{i=1}^{n} ru_{g\,i} + \sum_{i=1}^{n} sr_{g\,i} \rightarrow \lambda^{SR} \quad (9)$$

$$
\begin{aligned}
& R^{RU} + R^{SR} + R^{NS} \le \\
& \sum_{i=1}^{n} ru_{g\,i} + \sum_{i=1}^{n} sr_{g\,i} + \sum_{i=1}^{n} ns_{g\,i} \rightarrow \lambda^{NS}
\end{aligned} \quad (10)
$$

$$R^{RC} \le \sum_{i=1}^{n} rc_{g\,i} \rightarrow \lambda^{RC} \quad (11)$$

$$R^{RD} \le \sum_{i=1}^{n} rd_{g\,i} \rightarrow \lambda^{RD} \quad (12)$$

$$ru_{g\,i} \ge 0, sr_{g\,i} \ge 0, ns_{g\,i} \ge 0, rc_{g\,i} \ge 0, rd_{g\,i} \ge 0 \quad (13)$$

$$
\begin{aligned}
& ru_{g\,i} \le ru_i^{\max}, sr_{g\,i} \le sr_i^{\max}, ns_{g\,i} \le ns_i^{\max}, \\
& rc_{g\,i} \le rc_i^{\max}, rd_{g\,i} \le rd_i^{\max}
\end{aligned} \quad (14)
$$

Where, R^{RU}, R^{SR}, R^{NS} e R^{RC} e R^{RD} (in MW) are the required system amounts of each service during one hour. These quantities are considered known and defined by the ISO before the market auction is performed. Lambda variables are Lagrange multipliers associated to each constraint, and Upper limits in (14) are related to physical limits such as ramp rates and are part of the information of the reserve bids.

D. Bilateral Market Model

Private Bilateral contracts are considered as firm contracts which are authorized and implemented by ISO taking into consideration the conditions of the transmission network. In a compact form, bilateral contracts can be grouped in a GD matrix where each coefficient $GD_{i\,j}$ represents the MW traded between generators at bus *i* and load at bus *j*. Therefore, the total amount of contracts supplied by generator *i* is,

$$p_{g\,i}^{b} = \sum_{j=1}^{n} GD_{i\,j} \quad (15)$$

In addition, the total amount of contracts supplying the demand at bus *j* is,

$$p_{d\,j}^{b} = \sum_{i=1}^{n} GD_{i\,j} \tag{16}$$

In this model is supposed that firm long-term bilateral contracts have priority in relation to the pool dispatch. Because of this, the availability of generators participating in the pool market is constrained due to their already committed capacity with bilateral contracts as shown in (17).

$$p_{g\,i} \geq p_{g\,i}^{b} \tag{17}$$

E. Generator and Load Participations

In the integrated market, besides attending bilateral and pool loads, each generator *i* can participate bidding availability for several reserve services in the reserve market. The awarded reserve bids should respect the operational capacity limits of each generator as described in (18) and (19).

$$p_{g\,i} + ru_{g\,i} + sr_{g\,i} + ns_{g\,i} \leq p_{g\,i}^{\max} \tag{18}$$

$$-rd_{g\,i} + p_{g\,i} \geq p_{g\,i}^{b} \tag{19}$$

In addition, an important consideration is the capacity of response due to the technology employed by each generator *i* characterized by the corresponding ramp rate in MW/min. In the same way as it is done in [1], (20) shows how to consider this constraint in a time basis of 10 minutes through a linear relationship.

$$\frac{ru_{g\,i}}{RP_{g\,i}^{RU}} + \frac{rs_{g\,i} + ns_{g\,i}}{RP_{g\,i}^{SNS}} - 10 \leq 0 \tag{20}$$

F. Price of Reserve Services

The Lagrangean function of the optimization problem described in (1) to (20) allows to obtain expressions for the market clearing prices of services based on the dual variables as shown in (21) to (25).

$$\frac{\partial L}{\partial R^{RU}} = \lambda^{RU} + \lambda^{SR} + \lambda^{NS} = MCP_{RU} \tag{21}$$

$$\frac{\partial L}{\partial R^{SR}} = \lambda^{SR} + \lambda^{NS} = MCP_{SR} \tag{22}$$

$$\frac{\partial L}{\partial R^{NS}} = \lambda^{NS} = MCP_{NS} \tag{23}$$

$$\frac{\partial L}{\partial R^{RC}} = \lambda^{RC} = MCP_{RC} \tag{24}$$

$$\frac{\partial L}{\partial R^{RD}} = \lambda^{RD} = MCP_{RD} \tag{25}$$

Since Lagrange, multipliers are positive; this formulation ensures not reversal prices among services. In other words, $MCP_{RU} \geq MCP_{SR} \geq MCP_{NS}$ [1,6].

G. Pay as Bid pricing model

The pay as bid version of this model is implemented through an integration process based on the Auman-Shapley [1] technique that takes into consideration the non-linear characteristic existing in the transmission network due to transmission losses and voltage behaviors. These features are not present in simpler pay as bid schemes such as pro-rata. The integration process allows also to unbundling the use and prices of several services including pool dispatch, reserve availability and bilateral contracts. One-step of the process is described as follows.

Bilateral Contracts: Losses attributed to bilateral contracts due to the use of the transmission network are compensated in the Pool market. Initially, bilateral load is incremented by $dGD_{i\,j}$ while holding fixed the pool load and reserve requirements whose value at the very beginning are nil. In this step, the optimization problem minimizes costs of loss compensation due bilateral contracts and congestion management. The incremental problem to be solved is the following,

$$\underset{P_{g\,i}}{Min} \sum_{i=1}^{n} C_i(P_{g\,i}) \quad s.t. \quad \{ dGD \in S \cup E$$
$$and \quad (P_d^p, R^{RU}, R^{RG}, R^{NG}, R^{RC}, R^{RD}) \ fixed \} \tag{26}$$

If the solution of this problem is $dp_{g\,i}^{*}$ then the contract incremental losses are obtained through (27).

$$dp_{g\,i}^{bL} = dp_{g\,i}^{*} - \sum_{j=1}^{n} dGD_{i\,j} \tag{27}$$

Pool Market: In this step, the pool load is incremented by vector dp_d^p and reserve requirements are incremented sequentially by $dR^{RU}, dR^{SR}, dR^{NS}, dR^{RC}, dR^{RD}$ while holding fixed bilateral contracts load. The problem to be solved in this case is the following,

$$\underset{P_{g\,i}, ru_i, sr_i, ns_i, rc_i, rd_i}{Min} \sum_{i=1}^{n} [C_{g\,i}(p_{g\,i}) + C_{g\,i}^{RU}(ru_{g\,i}) + C_{g\,i}^{SR}(sr_{g\,i}) + C_{g\,i}^{NS}(ns_{g\,i}) + C_{g\,i}^{RC}(rc_{g\,i}) + C_{g\,i}^{RD}(rd_{g\,i})]$$
$$s.t. \quad \{(GD \ fixed), dP_d^p, dR^{RU}, dR^{SR}, dR^{NS}, dR^{RC}, dR^{RD}\} \in S \cup E \tag{28}$$

H. Integration Process

The integration process consists in solving alternatively problems (26) and (28) for small load increments following a uniform integration path from zero to the final value of the load, according to the "*t*" parameter such that *0 ≤ t ≤ 1*, as shown in (29).

$$X = \int_{t=0}^{1} dx(t) \tag{29}$$

Vector dX is composed of variables $dp_{g\,i}$, $dp^{bL}_{g\,i}$, $dru_{g\,i}$, $dsr_{g\,i}$, $dns_{g\,i}$, $drd_{g\,i}$ and $drc_{g\,i}$. In particular, the integration process of each load and required service follows the adjustment of parameters indicated in (30) to (32). Others reserve services follow the same procedure.

$$GD(t^b) = t^b \cdot GD, \quad 0 \le t^b \le 1 \tag{30}$$

$$P^p_d(t^p) = t^p \cdot P^p_d, \quad 0 \le t^p \le 1 \tag{31}$$

$$R^{RR}(t^{RU}) = t^{RU} \cdot R^{RU}, \quad 0 \le t^{RU} \le 1 \tag{32}$$

Considering the set of vectors $t = [t^b \,;\, t^p \,,\, t^{RU} \,,\, t^{SR} \,,\, t^{NS} \,,\, t^{RD}]^T$ of size $n \; x \; 1$ with $t^{(0)} = [0 \,;\, 0 \,,\, 0 \,,\, 0 \,,\, 0\,]^T$ and a constant increment step $dt = t/N$, the initial step of the integration process increases parameters sequentially as showed in s (33) to (35).

$$t^{(1)} = [dt^b, 0,0,0,0,0]^T \tag{33}$$

$$t^{(1)} = [dt^b, dt^p, 0,0,0,0]^T \tag{34}$$

$$t^{(1)} = [dt^b, dt^p, dt^{RU}, 0,0,0]^T \tag{35}$$

In the following integration steps, additional dt increments are added in the same order.

I. Revenues and Payments

This section presents how revenues and payments are obtained for generators and loads participating in the integrated market. These economic indexes are presented in incremental terms since they are calculated inside the integration process.

Bilateral Contracts

Because bilateral contracts are negotiated in private, their prices are not available. Nevertheless, we adopt as a price for these contracts the corresponding incremental costs of generators.

Revenues: The generator *i* revenue due to bilateral contracts and loss compensation is,

$$dc^b_{gi} = IC^b_{gi} \cdot \sum_{j=1}^{n} dGD_{i\,j} \tag{36}$$

Payments: Load *j* pays bilateral contracts according to (37).

$$dc^b_{d\,j} = \sum_{i=1}^{n} IC^b_i \cdot dGD_{i\,j} \tag{37}$$

Bilateral contracts should pay for losses and congestion management according to (38). This amount could be split among contract parties in a 50/50 basis or other proportion,

$$dc^{bL}_{i\,j} = \sum_{i=1}^{n} (\lambda_j - \lambda_i) \cdot dGD_{i\,j} \tag{38}$$

We adopt a split of 50/50 among contract parties. The corresponding payment of generator *i* for all its bilateral contracts is,

$$dc^{bcL}_{gi} = (1/2) \sum_{j=1}^{n} dc^{bL}_{i\,j} \tag{39}$$

Similarly, the payment of load *j* is,

$$dc^{bcL}_{d\,j} = (1/2) \sum_{i=1}^{n} dc^{bL}_{i\,j} \tag{40}$$

Pool and Reserve

Revenues: Generator *i* has a revenue portfolio from attending pool demand, bilateral loss compensation and congestion management as well as availability of reserve services given by (41) to (44).

$$dc^p_{g\,i} = \lambda_i \cdot dp^p_{g\,i} \tag{41}$$

$$dc^{bL}_{gi} = \lambda_i \cdot dp^{bL}_{g\,i} \tag{42}$$

$$dc^{RU}_{g\,i} = MCP_{RU} \cdot dru_{g\,i} \tag{43}$$

$$dc^{SR}_{g\,i} = MCP_{SR} \cdot dsr_{g\,i} \tag{44}$$

Incremental revenues of other reserve services follow similar calculations.

Payments: Load *j* pays pool demand and reserve services according to (43) to (45). In the case of reserve services, load *j* pays in a pro-rate manner.

$$dc^p_{d\,j} = \lambda_j dp^p_{d\,j} \tag{45}$$

$$dc^{RU}_{d\,j} = MCP_{RU} \cdot \left(\sum_{i=1}^{n} dru_{g\,i}\right) \cdot (p_{d\,j} / p_{d\,Total}) \tag{46}$$

$$dc^{SR}_{d\,j} = MCP_{SR} \cdot \left(\sum_{i=1}^{n} dsr_{g\,i}\right) \cdot (p_{d\,j} / p_{d\,Total}) \tag{47}$$

Incremental load payments related to the other services are obtained in a similar way.

At the end of the integration process by using (29), we have the net portfolio revenue of generator *i* which is composed basically by three terms corresponding to bilateral, pool and reserve markets as follows,

$$c_{gi} = c^b_{gi} + \hat{c}^p_{gi} + c^R_{gi} \tag{48}$$

Where,

$$c^R_{gi} = c^{RU}_{gi} + c^{RD}_{gi} + c^{SR}_{gi} + c^{NS}_{gi} + c^{RC}_{gi} \tag{49}$$

$$\hat{c}^p_{gi} = c^p_{g\,i} + c^{bL}_{gi} - c^{bcL}_{gi} \tag{50}$$

Likewise, the portfolio payment of load *j* is composed by three terms corresponding to bilateral, pool and reserve markets as follows,

$$c_{d\,j} = c^b_{d\,j} + c^p_{d\,j} + c^R_{d\,j} \tag{51}$$

Where,

$$c^R_{d\,j} = c^{RU}_{d\,j} + c^{RD}_{d\,j} + c^{SR}_{d\,j} + c^{NS}_{d\,j} + c^{RC}_{d\,j} \tag{52}$$

IV. RECONCILIATION OF COSTS

Under the pay-as-bid scheme, the costs allocated to the loads and bilateral contracts must match the generation cost components. This characteristic is verified at the end of the integration process by applying (29). Thus, for the supply of pool demand and associated loss and congestion management, we have,

$$\sum_{i=1}^{n} \hat{c}_{g\,i}^{\,p} = \sum_{j=1}^{n} c_{d\,j}^{\,p} \tag{53}$$

For services received by bilateral contracts from loss and congestion management, we have,

$$\sum_{i=1}^{n} c_{g\,i}^{bL} = \sum_{i=1}^{n}\ \sum_{j=1}^{n} c_{i\,j}^{bL} \tag{54}$$

For services supplying bilateral contracts, we have,

$$\sum_{i=1}^{n} c_{g\,i}^{b} = \sum_{j=1}^{n} c_{d\,j}^{b} \tag{55}$$

For all reserve services there is reconciliation of costs. As an example, (48) illustrate for the case of spinning reserve.

$$\sum_{i=1}^{n} c_{g\,i}^{SR} = \sum_{j=1}^{n} c_{d\,j}^{SR} \tag{56}$$

Reconciliation of costs is confirmed in the following numerical examples.

V. NUMERICAL RESULTS

This section analyzes results of an IEEE 5-Bus networks when the pricing model is applied. Table 1 shows the data used in the 5-Bus system. Per unit values are in the base of 100 MVA and 200 kV. Table 2 describes capacity limits of generators and the submitted bids in the energy market are characterized by the $c_{g\,i}$ continuous functions. In order to simplify the amount of data, the generators bid functions for each reserve service are considered half of the corresponding energy bid in Table 2 (i.e., $a_{g\,1}^{RU} = 10$ \$/MWh and $b_{g\,1}^{RU} = 0.02$ \$/MW^{2}h for generator 1 reserve bid and similar for the others generator regulation up reserve bids). Spinning reserve bids for all generators are considered higher and equal to the corresponding energy bids. The two reserve services required by the system are R^{RU}= 14.4 MW and R^{SR}= 40 MW corresponding to a total reserve margin of 5% of total load (i.,e., 54.4 MW). Substitution among these reserve services is allowed for supplying the total reserve requirement. In order to simplify the observation, there is no transmission congestion in the first two cases presented.

A total fixed system load of 1088 MW is considered and distributed in each bus according with the following vector: $p_d = [34{,}85{,}119{,}323{,}527]^T$ (MW). This load is supplied by bilateral and pool markets in different proportions in the next two examples. Table 3 shows bilateral contracts matrix GD. When the full amounts in GD matrix are implemented, they represent 94% of the total system load. In this case, the pool loads required at each bus are the coefficients of vector $p_d^p = [2{,}5{,}7{,}19{,}31]^T$ (MW). Bilateral tariffs are chosen by, $\pi_{i\,j}^{b} = dC_{g\,i}(P_{g\,i}^{b})\big/dP_{g\,i}$ for all j.

TABLE 1. NETWORK DATA OF 5-BUS SYSTEM

From Bus	To Bus	r (pu)	x (pu)	b (pu)	$p_{flow}^{\max}$ (pu)
1	2	0,0147	0,168	0,138	3,00
1	4	0,0108	0,126	0,102	3,55
2	3	0,0185	0,210	0,185	3,00
3	4	0,0294	0,336	0,296	3,00
3	5	0,0221	0,252	0,213	3,00
4	5	0,0108	0,126	0,104	4,50
2	4	0,0105	0,130	0,100	3,60

TABLE 2. GENERATORS ENERGY BIDS AND CAPACITY

$$C_{g\,i}(p_{g\,i}) = c_{0\,i} + a_i \cdot p_{g\,i} + 0.5 \cdot b_i \cdot p_{g\,i}^2 \quad (\$/h)$$

Bus	$p_g^{\min}$ (MW)	$p_g^{\max}$ (MW)	c_0 (\$/h)	a (\$/MWh)	b (\$/MW^{2}h)
1	0	460	0	20	0.040
2	0	500	0	21	0.030
3	0	500	0	25	0.045
4	0	500	0	56	0.040
5	0	500	0	57	0.040

TABLE 3. BILATERAL CONTRACTS (MW)

Bus		Loads (MW)					
		1	2	3	4	5	Total
Generators (MW)	1	32	48	32	144	160	416
	2	0	32	32	112	240	416
	3	0	0	48	48	96	192
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	Total	32	80	112	304	496	

Tables 4, 5 and 6 show results for a base case without transmission congestion considering that 47% of total load is supplied by the bilateral contract market. In this case, total losses are equal to 42.86 MW. Table 4 shows the integrated market dispatch and prices. The more economical operation forces generators 1 and 2 to participate in all markets. Incremental costs for energy are about the same in the three generators participating in this market. Small differences are due to transmission losses. Incremental costs for reserve of generators 1 and 2 are the about the same and equal to 10.76\$/MWh and 10.75\$/MWh. The difference is because generator 2 capacity is fully committed. Average prices received by generators are smaller than marginal prices. The MCP$_{ru}$ is 10.8\$/MWh in the reserve market which is a little higher than the corresponding average prices. Tables 5 and 6 show portfolios of generators revenues and loads payments. These tables show in detail each revenue component for generators and payment components for loads. Generator at bus 1 and 2 are the only to obtain revenues from the three products. The reconciliation of revenues and payments for energy (bilateral and pool) and reserve can be observed by comparing the last rows of both tables and consequently no Merchandising Surplus (MS) is produced (see total revenues and total payments in these two tables). If the same generator bids were used in a marginal pricing approach, the total cost of operation is 41038 \$/h which represents an increase of 24.3% in relation to the total cost obtained by PAB.

TABLE 4. ENERGY AND RESERVE MARKET DISPATCH THROUGH THE INTEGRATED PAB APPROACH. BILATERAL CONTRACTS PARTICIPATION EQUAL TO 47% OF TOTAL LOAD. POWER IN MW, λ, π_g^b AND AVERAGE COSTS IN \$/MWH. ROWS WITH * CONTAIN GIVEN DATA

Bus	1	2	3	4	5	Total
P_g^b	208.0	208.0	96.0	0.0	0.0	512.0
P_g^p	174.8	275.8	168.5	0.0	0.0	618.9
P_g	382.8	483.6	264.5	0.0	0.0	1130.9
ru_g	38	16.4	-0.0	-0.0	-0.0	54.4
sr_g	0.0	-0.0	0.0	-0.0	-0.0	-0.0
Total R	38	16.4	-0.0	-0.0	-0.0	54.4
$*P_d^b$	16.0	40.0	56.0	152.0	248.0	512.0
$*P_d^p$	18.0	45.0	63.0	171.0	279.0	576.0
$*P_d$	34.0	85.0	119.0	323.0	527.0	1088.0
λ	35.3	35.5	36.9	37.8	40.6	
IC_g	35.3	35.5	36.9	56.0	57.0	
$*\pi_g^b$	28.3	27.2	29.3	56.0	57.0	
C_g^b/P_g^b	24.2	24.1	27.2			
C_g^p/P_g^p	31.8	31.4	33.1			
C_g^{RU}/ru_g	10.5	10.6				

TABLE 5. INTEGRATED PAY-AS-BID APPROACH
GENERATORS REVENUE PORTFOLIO (\$/H)

Bus	Energy		Reserve	Total
	c_g^b	$\hat{c}_g^p$	c_g^R	
1	5025.3	5560.4	394.1	10979.8
2	5017.0	8645.4	174.6	13837.0
3	2607.4	5579.9	0.0	8187.3
4 and 5	0.00	0.00	0.0	0
Total	12649.6	19785.7	568.7	33004.0

TABLE 6. INTEGRATED PAY-AS-BID APPROACH.
LOADS PAYMENT PORTFOLIO (\$/H)

Bus	Energy		Reserve	Total
	c_d^b	c_d^p	c_d^R	
1	395.3	618.3	17.8	1031,4
2	988.2	1545.8	44.4	2578,4
3	1383.6	2164.1	62.2	3609,9
4	3755.3	5873.9	168.8	9798
5	6127.1	9583.7	275.4	15986,2
Total	12649.6	19785.7	568.7	33004

As a second example, we increase the total bilateral contract participation from 47% to 94% of the same total load (i.e., pool load participation is decreased) without transmission congestion. Table 7 shows that generators 1 and 2 are heavily committed with bilateral contracts (83% of their capacity). Total losses are equal to 43.63 MW in this case. At this level of generation, only the low cost generator 2 participates in the three markets. The energy incremental costs of active generators are the almost the same and small differences are due to transmission losses. With this level of committed capacity due to the contracts, it is more economical for the system not award the bid energy pool of generator 1 and only accepts the bid energy pool of generator 2. What is interesting here compared with the previous case is that the two low bids for regulation up of generator 1 and generator 2 are awarded with 37.6 MW and 16.8MW giving the same incremental cost of 10.8\$/MWh in the reserve market (in the marginal case, MCP_{ru}=10.8\$/MWh). This fact is due to the freedom opened by the small pool market. Reserve average prices charged by generators are almost the same as in the previous case and lower than the marginal price for reserve. As can be seen comparing Tables 8 and 9, no merchandising surplus is produced. Moreover, reconciliation of revenues and payments among generators and loads are verified in both markets (see the last rows of these two tables). Total payments and revenues increase 30\$/h in relation to the previous case due to the out of merit operation imposed by the strong bilateral presense. If the same generator bids were used in a marginal pricing approach, total cost of operation is 40296.1 \$/h which represents an increase of 22% in relation to the total cost obtained by PAB.

TABLE 7. ENERGY AND RESERVE MARKET DISPATCH THROUGH THE INTEGRATED PAB APPROACH. BILATERAL CONTRACTS PARTICIPATION EQUAL TO 94% OF TOTAL LOAD. POWER IN MW, λ, π_g^b AND AVERAGE COSTS IN \$/MWH. ROWS WITH * CONTAIN GIVEN DATA

Bus	1	2	3	4	5	Total
P_g^b	416.0	416.0	192.0	0.0	0.0	1024.0
P_g^p	0.0	47.8	59.8	0.0	0.0	107.6
P_g	416.0	463.8	251.8	0.0	0.0	1131.6
ru_g	37.6	16.8	-0.0	-0.0	-0.0	54.4
sr_g	-0.0	-0.0	0.0	-0.0	-0.0	-0.0
Total R	37.6	16.8	-0.0	-0.0	-0.0	54.4
$*P_d^b$	32.0	80.0	112.0	304.0	496.0	1024.0
$*P_d^p$	2.0	5.0	7.0	19.0	31.0	64.0
$*P_d$	34.0	85.0	119.0	323.0	527.0	1088.0
λ	34.5	34.9	36.3	37.1	39.9	-
IC_g	36.6	34.9	36.3	56.0	57.0	-
$*\pi_g^b$	36.6	33.5	33.6	56.0	57.0-	-
C_g^b/P_g^b	28.3	27.2	29.3	-	-	-
C_g^p/P_g^p	-	34.2	35	-	-	-
C_g^{RU}/ru_g	10.4	10.6	-	-	-	-

Other simulations where performed by varying the total reserve requirement in both of the previous cases. For instance, when the system reserve requirement is increased to 10% of total load (108MW), in the first case (with 47% of bilateral contracts) only generators 1 and 2 participate in three markets with full capacity. In the reserve market, generator 1 has a participation of 76.5MW with an average price of 10.8\$/MWh and generator 2 has a participation of 32.3MW with an average price of 10.7\$/MWh. On the other hand, the corresponding marginal price MCP_{ru} is equal to 12.2\$/MWh. In the second case the difference between average and marginal prices is about the same. For more than 10% of system reserve requirement both average prices and marginal price increase but always average prices stay lower than marginal price charged by generators. Because of space reasons we do not added and discuss average prices paid by loads.

TABLE 8. INTEGRATED PAY-AS-BID APPROACH
GENERATORS REVENUE PORTFOLIO (\$/H)

Bus	Energy		Reserve	Total
	c_g^b	c_g^p	c_g^R	
1	11781.1	0.0	390.1	12171.3
2	11331.8	1636.0	178.5	13146.4
3	5629.4	2091.9	0.0	7721.3
4 and 5	0.0	0.0	0.0	0.0
Total	28742.4	3727.9	568.7	33039.0

TABLE 9. INTEGRATED PAY-AS-BID APPROACH.
LOADS PAYMENT PORTFOLIO (\$/H)

Bus	Energy		Reserve	Total
	c_d^b	c_d^p	c_d^R	
1	898.2	116.5	17.8	1032.5
2	2245.5	291.2	44.4	2581.2
3	3143.7	407.7	62.2	3613.6
4	8532.9	1106.7	168.8	9808.4
5	13922.1	1805.7	275.4	16003.3
Total	28742.4	3727.9	568.7	33039.0

The previous numerical cases show that the suggested integrated model under pay-as-bid approach reserve amounts in MW and their corresponding average prices are stable and almost independent from proportions of bilateral and pool markets. This is an important economic signal for reserve market as well as for the energy market.

VI. CONCLUSION

In this paper a new pricing model is presented with the following characteristics: i) incorporation of bilateral, pool, and reserve markets in a joint market of services; ii) integration allows to know the impact of markets interactions on the operation and consequently on prices; iii) Allows the possibility of assessment of pay as bid pricing against the uniform pricing approaches; iv) Allows to produce detailed portfolios for agents in terms of revenues and payments for each awarded bid in each market; v) Allows the possibility of testing several operating scenarios with price strategies in order to evaluate the impact on agents portfolios.

The model considers the transmission system operation in detail including generation and transmission capacities, transmission losses, voltage limits and reactive limits. Bilateral contracts are treated as physical firm contracts loading transmission lines and therefore producing transmission losses. Several reserve services are considered according with the quality of speed response. The possibility of substitution among these services is allowed for cost reduction without creating the undesirable price reversal.

The characteristics of this model make it attractive for analyzing the impact of several operation scenarios and bid strategies on the agent's portfolios. Further research is under development for studying the impact of transmission congestion on reserve prices in the integrated market and application on large power systems.

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